NABBEM05E.2014-2015.1A Exam November 4 2014

Electricity and Magnetism 2
Instructor: A.M. van den Berg

You don't have to use separate sheets for every question.
Write your name and S number on every sheet
There are **6 questions** with a total number of marks: 100

WRITE CLEARLY

(1) (Total 12 marks)

(i) (2 marks)
In electrodynamics, what can give rise to a curling magnetic field $\vec{H}$?
(a) a bound (magnetization) current density $\vec{J}_M$
(b) a free current density $\vec{J}$
(c) a free current density $\vec{J}$ and a time-varying electric field $\vec{D}$
(d) stationary electric charges

(ii) (2 marks)
What are the boundary conditions that the electric fields obey on the surface between two materials when there are no free charges on the surface?
(a) both $\vec{D}$ and $\vec{E}$ approach infinity at boundary surfaces
(b) the field $\vec{D}$ is continuous and the curl of $\vec{E}$ is continuous
(c) the normal components of $\vec{E}$ are continuous and the tangential components of $\vec{D}$ are continuous
(d) the normal components of $\vec{D}$ are continuous and the tangential components of $\vec{E}$ are continuous

(iii) (2 marks)
If a system could be built where a time-varying electric field $\vec{E}$ is always parallel to a time varying magnetic field $\vec{H}$ at every point in space, what would be the nature of the electromagnetic energy flow?
(a) energy would flow parallel to the $\vec{E}$ field, but in the opposite direction
(b) energy would flow parallel to the $\vec{E}$ field, but in the same direction
(c) energy would flow perpendicular to the $\vec{E}$ field
(d) there would be no energy flow

(iv) (2 marks)
A loop of conducting wire sits in a fixed magnetic field. In general, what would moving the wire do?
(a) nothing
(b) induce a current in the wire
(c) induce an electric dipole moment in the wire
(d) freeze the wire

(v) (2 marks)
A permanent bar magnet is in the shape of a long cylindrical rod and contains a

constant, uniform magnetization that points along the cylinder's axis. Where is the bound (magnetization) current $\vec{J}_M$ found?

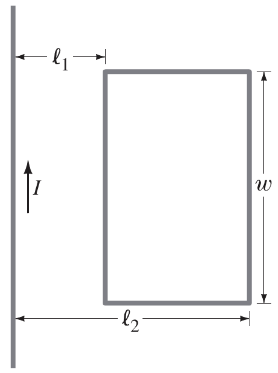
(a) flowing around the round sides of the cylindrical rod
(b) flowing in little circles on the flat top and bottom ends of the cylindrical rod
(c) flowing in little circles everywhere inside the rod
(d) there are no currents

(vi) (2 marks)
If magnetic monopoles did exist, how many of Maxwell's equations would have to be changed?

(a) none
(b) one
(c) two
(d) all of them

(2) (Total 10 marks)

A long straight wire and a small rectangular wire loop lie in the same plane. Determine the mutual inductance in terms of ℓ_1, ℓ_2, and w. Assume that the wire is very long compared to ℓ_1, ℓ_2, and w.



Perform the following steps. First calculate the magnetic field induced by the long wire as a function of the radial distance s. Then determine the flux of the magnetic field through the rectangular wire loop. Finally, determine the mutual inductance.

(3) (Total 25 marks)

A parallel-plate capacitor consists of two circular plates of area $A = 5\ \text{cm}^2$ with a mutual separation distance of 2.0 mm. The plates are in vacuum. The charging current I_C has a constant value of 1.80 mA. At $t = 0$ the charge on the plates is $Q_o = 0$ Coulomb.

(a) (5 marks)
Calculate the charge Q on the plates, the electric field E between the plates, and the potential difference V between the plates when $t = 0.5\ \mu$s.

(b) (5 marks)
Calculate dE/dt, the time rate of change of the electric field E between the plates. Does dE/dt vary in time?

(c) (5 marks)
Calculate the displacement current-density J_D between the plates and from this the displacement current I_D. How do I_C and I_D compare?

(d) (5 marks)
Using Ampère's law, what is the induced magnetic field strength B between the plates at a distance of $r = 2.0$ cm from the axis?

(e) (5 marks)
What is the induced magnetic field strength B between the plates at a distance of $r = 1.0$ cm from the axis? And at 0.0 cm from the axis?

(4) (Total 30 marks)

An electromagnetic plane wave of (angular) frequency ω is traveling in the x-direction through the vacuum. It is polarized in the y-direction, and the amplitude of the electric field is E_o.

(a) (5 marks)
Write down the electric and magnetic fields, $E(x, y, z, t)$ and $B(x, y, z, t)$. Be sure to explain any auxiliary quantities you introduce in terms of ω, E_o, and the constants of nature.

(b) (5 marks)
This same wave is observed from an inertial frame $\tilde{S}$ moving in the x direction. Find the electric and magnetic fields in $\tilde{S}$, and express them in terms of the $\tilde{S}$ coordinates: $\tilde{E}(x, y, z, t)$ and $\tilde{B}(x, y, z, t)$.

(c) (5 marks)
What is the frequency $\tilde{\omega}$ of the wave in $\tilde{S}$?

(d) (5 marks)
From $\tilde{\omega}$ and $\tilde{\lambda}$, determine the speed of the wave in $\tilde{S}$. Is it what you expected?

(e) (5 marks)
What is the ratio of the intensity in $\tilde{S}$ to the intensity in S?

(f) (5 marks)
As a youth, Einstein wondered what an electromagnetic wave would look like if you could run along beside it at the speed of light. What can you tell him about the amplitude, frequency, and intensity of the wave, as v approaches c?

The relevant Lorentz transformations are:

$$\tilde{E}_x = E_x \qquad \tilde{E}_y = \gamma(E_y - vB_z) \qquad \tilde{E}_z = \gamma(E_z + vB_y)$$

$$\tilde{B}_x = B_x \qquad \tilde{B}_y = \gamma(B_y + \frac{v}{c^2}E_z) \qquad \tilde{B}_z = \gamma(B_z - \frac{v}{c^2}E_y)$$

$$x = \gamma(\tilde{x} + v\tilde{t}) \qquad y = \tilde{y} \qquad z = \tilde{z}$$

$$t = \gamma(\tilde{t} + \frac{v}{c^2}\tilde{x})$$

(5) (Total 10 marks)

Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_o, frequency ω, and phase angle zero that is

(a) (5 marks)
traveling in the negative x direction and polarized in the z direction.

(b) (5 marks)
traveling in the direction from the origin to the point (1,1,1), with polarization parallel to the xz plane.

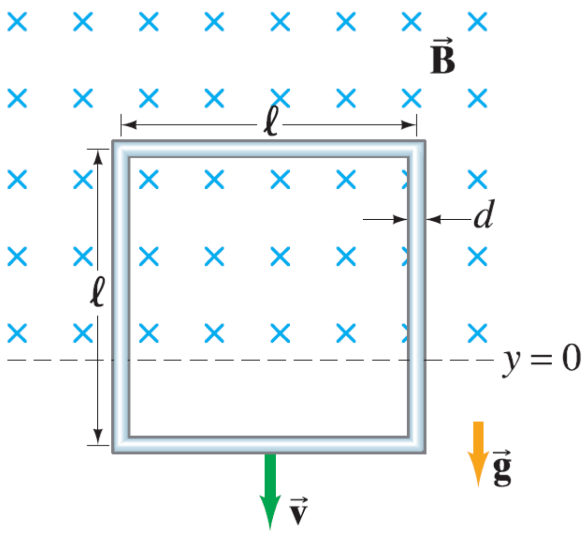
In each case, sketch the wave, and give the explicit Cartesian components of $\vec{k}$ and $\hat{n}$.

(6) (Total 13 marks)

In a certain region of space near Earth's surface, a uniform horizontal magnetic field of magnitude B exists above a level defined to be $y = 0$. Below $y = 0$, the field abruptly becomes zero, see the figure. A vertical square wire loop has a resistivity ρ, mass density ρ_m, diameter d, and site length ℓ. It is initially at rest with its lower horizontal side at $y = 0$ and then is allowed to fall under gravity, with its plane perpendicular to the direction of the magnetic field.

(a) (7 marks)
While the loop is still partially immersed in the magnetic field (as it falls into the zero-field region), determine the magnetic "drag" force that acts on it at the moment when its speed is v.

(b) (6 marks)
Assume that the loop achieves a terminal velocity v_T before its upper horizontal side exits the field. Determine a formula for v_T.



1 i c Maxwell 4

ii d eqs 7.60 and 7.62

iii d $\vec{S} = \vec{E} \times \vec{B} = 0$

iv b only on very specific cases $I = 0$.

v a eqs 7.48

vi c Maxwell 2 and 3

2 $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

long straight wire: $2\pi s B = \mu_0 I$

$B = \mu_0 I / (2\pi s)$.

$$\vec{\Phi}_B = \int_S \vec{B} \cdot d\vec{a} = \int_0^w \int_{l_1}^{l_2} \frac{\mu_0 I}{2\pi s} \, ds \, dz$$

$$= \frac{w \mu_0 I}{2\pi} \int_{l_1}^{l_2} \frac{ds}{s}$$

$$= \frac{w \mu_0 I}{2\pi} \ln\left(\frac{l_2}{l_1}\right)$$

$$M = \frac{\Phi_B}{I} = \frac{w \mu_0}{2\pi} \ln\left(\frac{l_2}{l_1}\right)$$

3a $Q = I . t = 1.8\, 10^{-3}\ 5 . 10^{-7} = 9 . 10^{-10}\, C.$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0} = \frac{9 . 10^{-10} / 5 . 10^{-4}}{8{,}85 . 10^{-12}}$$

$$= 2{,}034\ 10^{5}\ V/m$$

$$V = E d = 2{,}034\ 10^{5} * 2 . 10^{-3} = 406.8\ V.$$

b $$\frac{dE}{dt} = \frac{dQ}{dt}\frac{1}{A\varepsilon_0} = \frac{I}{A\varepsilon_0} = \frac{1.8\, 10^{-3}}{5 . 10^{-4}\ 8{,}85\ 10^{-12}}$$

$$= 4{,}07\ 10^{11}\ V/(m s)$$

I = constant, therefore $\frac{dE}{dt}$ = constant

c $$J_D = \varepsilon_0 \frac{dE}{dt} = \varepsilon_0 \frac{I}{\varepsilon_0 A} = \frac{I}{A} = \frac{1.8\, 10^{-3}}{5 . 10^{-4}} = 3.6\ A/$$

$$\left.\begin{array}{l} I_D = J_D A = 1{,}8 . 10^{-3}\ A \\ I_C = 1{,}8 . 10^{-3}\ A \end{array}\right\} \text{equal}$$

3 d.

The radius of one plate r is

$$r = \left(\frac{A}{\pi}\right)^{1/2} = 1.26\ 10^{-2}\ m.$$

$$\oint \vec{B}\,d\vec{l} = \mu_0 I_{encl}$$

I_{encl} for $r = 2.0\,cm$ is equal to the charging current.

$$I_{encl} = 1.8\ 10^{-3}\ A$$

$$B\ 2\pi s = 4\pi.\ 10^{-7}\ 1.8\ 10^{-3}$$

$$B = \frac{4\pi.\ 10^{-7}\ 1.8\ 10^{-3}}{2\pi\ 2.\ 10^{-2}} = 1.8.\ 10^{-8}\ T.$$

e. In this case the enclosed current is the current density J_D between the plates integrated over an area with radius 1cm.

$$B = \frac{\mu_0 J_D (\pi s^2)}{2\pi s} = \frac{\mu_0 J_D s}{2} = 2.25\ 10^{-8}\ T.$$

for $r = 1$

For $r = 0$ $B = 0$ (no enclosed current)

4a

$$E(x,y,z,t) = E_0 \cos(kx - \omega t)\,\hat{y}$$

because polarization is in $\hat{y}$ direction, so must $\vec{E}$ be in $\hat{y}$ direction
Sign of ω (and/or) k must be such that wave moves onto x direction; therefore argument is $(kx - \omega t)$ and not $(kx + \omega t)$

$$B(x,y,z,t) = \frac{E_0}{c} \cos(kx - \omega t)\,\hat{z}$$

Amplitude of $B = \frac{E_0}{c}$, direction must be $\hat{z}$ because $\vec{E} \wedge \vec{B}$ provides direction of traveling
$\vec{B}$ and $\vec{E}$ must be in phase with each other.

$$k = \frac{\omega}{c}$$

b $\tilde{E}_x = \tilde{E}_z = 0 \qquad \hat{B}_x = \tilde{B}_y = 0.$

$$\tilde{E}_y = \gamma(E_y - vB_z) =$$

$$= \gamma E_0 \left[1 - \frac{v}{c}\right] \cos(kx - \omega t)$$

$$= \alpha E_0 \cos(kx - \omega t) \qquad \alpha = \gamma\left(1 - \frac{v}{c}\right)$$

$$\tilde{B}_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right) = \gamma E_0 \left[\frac{1}{c} - \frac{v}{c^2}\right] \cos(kx - \omega t)$$

$$= \frac{\alpha}{c} E_0 \cos(kx - \omega t)$$

4 b continued

Now write x and t in $\tilde{S}$ frame

$$kx - \omega t = \gamma\left[k(\tilde{x} + v\tilde{t}) - \omega\left(\tilde{t} + \frac{v\tilde{x}}{c^2}\right)\right]$$

$$= \gamma\left[\left(k - \frac{\omega v}{c^2}\right)\tilde{x} - (\omega - kv)\tilde{t}\right]$$

$$= \tilde{k}\tilde{x} - \tilde{\omega}\tilde{t}$$

with $\tilde{k} = \gamma\left(k - \frac{\omega v}{c^2}\right) = \gamma k\left(1 - \frac{v}{c}\right)$

$$= \alpha k$$

and $\tilde{\omega} = \gamma(\omega - kv) = \gamma\left(1 - \frac{kv}{\omega}\right)\omega$

$$\tilde{\omega} = \gamma\left(1 - \frac{v}{c}\right)\omega = \alpha\omega.$$

Therefore.

$$\tilde{E}(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{t}) = \tilde{E}_0 \cos(\tilde{k}\tilde{x} - \tilde{\omega}\tilde{t})\,\hat{y}$$

$$\tilde{B}(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{t}) = \frac{\tilde{E}_0}{c} \cos(\tilde{k}\tilde{x} - \tilde{\omega}\tilde{t})\,\hat{z}$$

with $\tilde{E}_0 = \alpha E_0$

$$\tilde{k} = \alpha k$$

$$\tilde{\omega} = \alpha\omega$$

with $\alpha = \frac{(1 - v/c)^{1/2}}{(1 + v/c)^{1/2}}$

4c. $$\tilde{\omega} = \alpha \omega = \gamma\left(1 - \frac{v}{c}\right)\omega$$

$$= \frac{\left(1 - \frac{v}{c}\right)}{\left(1 - \frac{v}{c}\right)^{1/2}\left(1 + \frac{v}{c}\right)^{1/2}}\,\omega$$

$$= \frac{\left(1 - v/c\right)^{1/2}}{\left(1 + v/c\right)^{1/2}}\,\omega$$

This is doppler shift.

4d. $$\tilde{\lambda} = \frac{2\pi}{\tilde{k}} = \frac{2\pi}{\alpha k} = \frac{\lambda}{\alpha}$$

$$\tilde{V}_{EM} = \frac{\tilde{\omega}}{\tilde{k}} = \frac{\alpha \omega}{\alpha k} = \frac{\omega}{k} = c$$

this is what I expect from Einstein.

4e. $$\frac{\tilde{I}}{I} = \frac{\tilde{E}_0^{2}}{E_0^{2}} = \alpha^{2} = \frac{1 - v/c}{1 + v/c}$$

4f Everything goes to zero

5a.

Monochromatic ω is fixed

$\delta = 0$

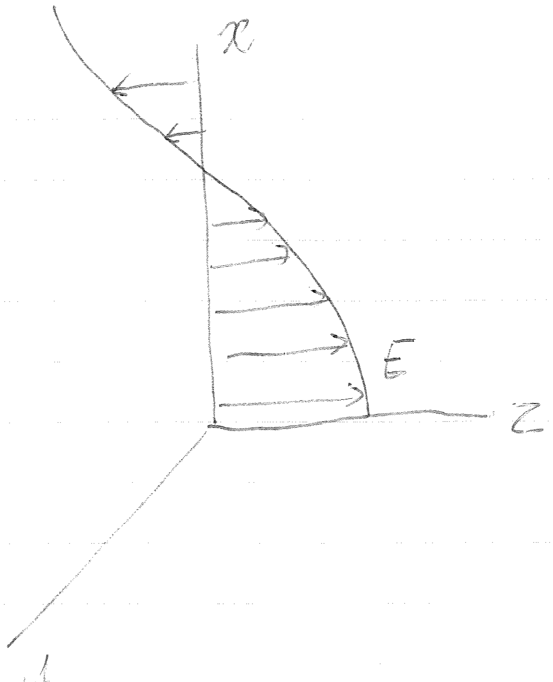
negative x direction; argument is $(kx + \omega t)$

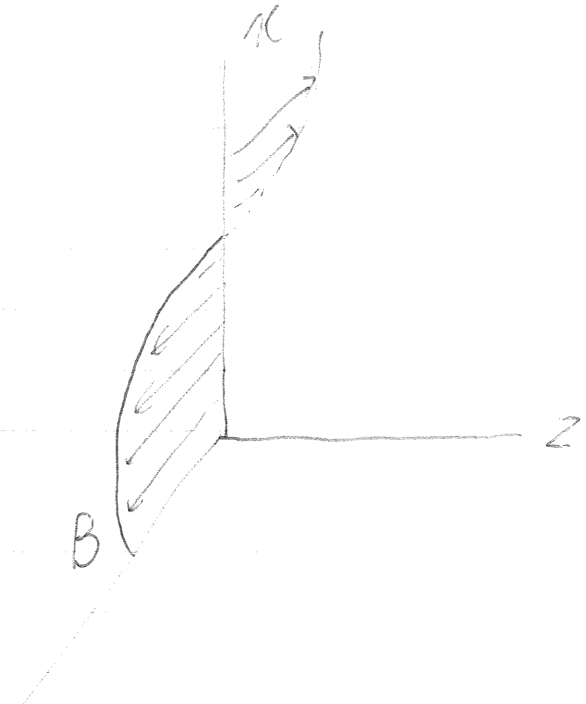
Polarized z-direction $\vec{E}$ has only a z component

B component is $\hat{k} \times \hat{n} = -\hat{x} \times \hat{z} = +\hat{y}$

$$\vec{E}(x,t) = E_0 \cos(kx + \omega t)\,\hat{z}$$

$$B(x,t) = \frac{E_0}{c} \cos(kx + \omega t)\,\hat{y}$$





5 b

The wave travels from (0,0,0) to (1,1,1)
Therefore

$$\vec{k} = |k| \left(\frac{\hat{x} + \hat{y} + \hat{z}}{\sqrt{3}} \right)$$

Polarization is in xz plane,
thus $\hat{n} = \frac{\hat{x} - \hat{z}}{\sqrt{2}}$ (or $\frac{\hat{x} + \hat{z}}{\sqrt{2}}$)

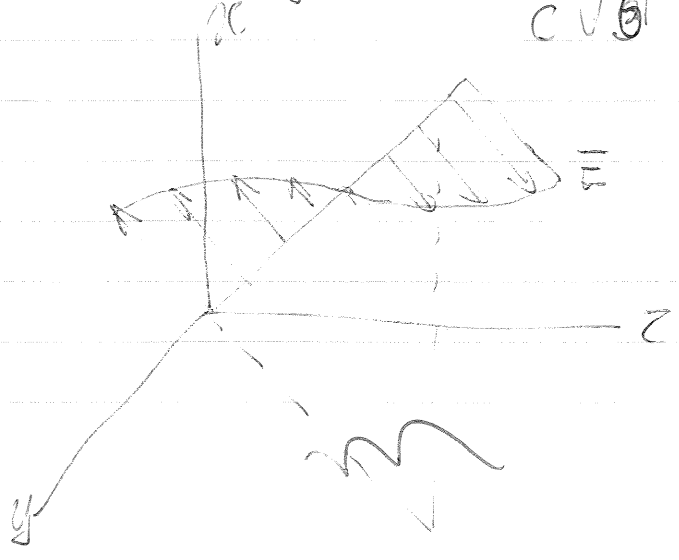
However $\hat{n} \cdot \vec{k} = 0$
means $\hat{n} \cdot \vec{k} = \frac{1}{\sqrt{6}} (\hat{x}^2 \pm \hat{z}^2)$

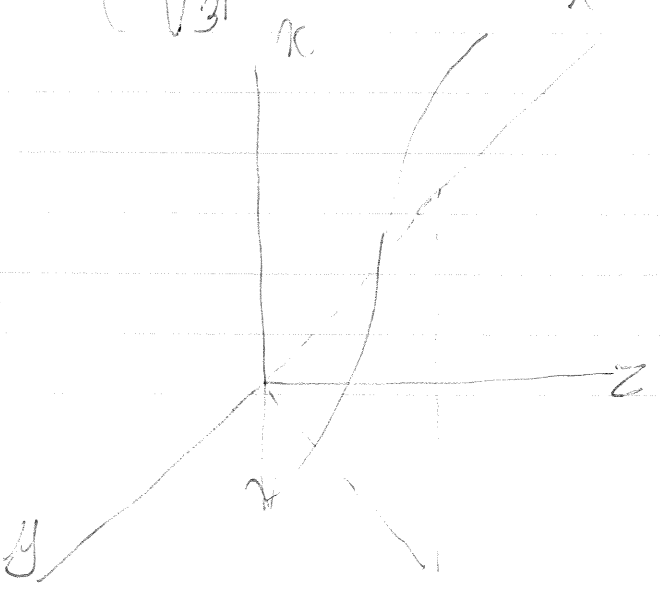
$= \frac{1}{\sqrt{6}} (1 \mp 1) = 0$

only for $n = \frac{\hat{x} - \hat{z}}{\sqrt{2}}$ and $\vec{k} \times \vec{n} = \frac{1}{\sqrt{6}} (-\hat{x} + 2\hat{y} - \hat{z})$

$$E(x, y, t) = \frac{E_0}{\cancel{\sqrt{3}}} \cos \left(\frac{|k|}{\sqrt{3}} (x + y + z) - \omega t \right) \frac{\hat{x} - \hat{z}}{\sqrt{2}}$$

$$B(x, y, t) = \frac{E_0}{c\sqrt{6}} \cos \left(\frac{|k|}{\sqrt{3}} (x + y + z) - \omega t \right) (-\hat{x} + 2\hat{y} - \hat{z})$$





6 a.

$$\mathcal{E} = -\frac{d\phi_B}{dt} = -\frac{B\,dA}{dt} = -Blv.$$

$$I = \frac{\mathcal{E}}{R} \qquad \rho_e = \text{resistivity}$$
$$\rho_m = \text{specific density}$$

$$R = \frac{L}{d}\rho_e = \frac{4l}{\pi d^2/4}\rho_e = \frac{16\rho_e l}{\pi d^2}$$

$$I = -Blv\left(\frac{16\rho_e l}{\pi d^2}\right)^{-1}$$

$$F_m = I\,l\,B = \frac{\pi d^2}{16\rho l} Blv\,lB$$

$$= \frac{\pi d^2 B^2 lv}{16\rho_e}$$

b $$F_g = mg = \left(4\pi l \frac{d^2}{4}\right)\rho_m g$$

$$F_g = F_m \Rightarrow \frac{\cancel{\pi} \cancel{d^2} B^2 \cancel{l} v}{16\rho_e} = \frac{4\cancel{\pi}\cancel{l}\cancel{d^2}}{\cancel{4}}\rho_m g$$

$$v = \frac{16\rho_e\rho_m g}{B^2}$$